Base 2 Basics

Place value in the binary system is just like place value in the familiar decimal system, except that instead of 10 digits (0,1,2,3,4,5,6,7,8 and 9), there are only 2 (0 and 1). ("Binary digit" is sometimes abbreviated as "bit").

In a whole decimal number: the rightmost place is the 1's place;
to the left of it is the 10's place;
next is the 100's place (because $100 = 10 \times 10 = 10^2$);
then the 1000's place (because $1000 = 10 \times 10 \times 10 = 10^3$); and so on.

eg. $609 = 6 \times 100 + 0 \times 10 + 9 \times 1 = 600 + 9 = 609$

So in a whole binary number: the rightmost place is the 1's place;
to the left of it is the 2's place;
next is the 4's place (because $4 = 2 \times 2 = 2^2$);
then the 8's place (because $8 = 2 \times 2 \times 2 = 2^3$); and so on.

eg. $1010 = 1 \times 8 + 0 \times 4 + 1 \times 2 + 0 \times 1 = 8 + 2 = 10$

To avoid confusion between bases, subscripts can be used. Thus, the result of the previous line could be written as $1010_2 = 10_{10}$.

Now, of the 10 types of people in the world – those who understand binary and those who don't – which are you?

But what about fractions and decimals?

To the right of the decimal point in a decimal number, the place values are negative powers of 10.
The first place after the decimal point is the $\frac{1}{10}$'s place (because $\frac{1}{10} = 10^{-1}$),
The next place is the $\frac{1}{100}$'s place (because $\frac{1}{100} = 10^{-2}$), and so on.

For a binary number, we could call the point a binary point.
To the right of the point in a binary number, the place values are negative powers of 2.
The first place after the point is the $\frac{1}{2}$'s place (because $\frac{1}{2} = 2^{-1}$),
The next place is the $\frac{1}{4}$'s place (because $\frac{1}{4} = 2^{-2}$), and so on.

So, for example, $.11 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = .75$, and $.0101\overline{01} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots = \frac{1}{3} = .33\overline{3}$

Rational numbers always have repeating patterns in their expansions, no matter what the base is.

But what about irrational numbers?

Irrational numbers have expansions that go on forever, never culminating in a repeating pattern.

For example, first note that $11.001001 = 2 + 1 + \frac{1}{8} + \frac{1}{64} \approx 3.14$
and then 11.0010010000111111

$$= 2 + 1 + \frac{1}{8} + \frac{1}{64} + \frac{1}{2048} + \frac{1}{4096} + \frac{1}{8192} + \frac{1}{16384} + \frac{1}{32768} + \frac{1}{65536}$$

$$\approx 3.14159$$

Finally, $11.001001000011111101\ldots = 3.14159\ldots = \pi$

So your new t-shirt says: "***Binary is as easy as pi***".
It is, isn't it?

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